RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2014

THIRD YEAR

Date : 20/12/2014 **PHYSICS (Honours)**

Time : 11 am – 1 pm **Paper :** V (Gr. B) Full Marks : 50

Unit – I

(*Answer **any three** questions*) [3×10]

1. a) Consider a one dimensional quantum Harmonic Oscillator. Taking the creation and annihilation operators a and a^+, construct a Unitary operator out of them. [3]

   b) Consider a double-slit experiment for electrons, with slits A and B. If $\psi_A(y)$ is the amplitude for detecting the electron at the point y on the screen, if it goes through slit A, $\psi_B(y)$ the amplitude if it goes through B, what is the probability amplitude of detecting an electron at y? Write the principle you utilise here. [2]

   c) Write down the time dependent Schröclinger equation. Use it to deduce the equation of continuity. Give the physical interpretation of the wave function. [5]

2. Calculate the reflection and transition amplitudes for a particle moving from x < 0 right towards a potential barrier given by

   $V(x) = 0, x < 0$

   $= V_0, x \in (0, a),\ V_0 \sim \text{const}$

   $= 0, x > a$

   Do the calculation both for E < V and E > V. [5+5]

3. a) What is a linear operator? Discuss the relevance of this operator in he content of quantum mechanics. Show that $x\frac{d}{dx}$ is a linear operator. [4]

   b) Prove that the linear combination of two eigen functions with two different eigen values cannot be an eigen function of an operator. [1]

   c) Calculate the probability current density and interpret the result for the wave function $\psi = \frac{1}{r}e^{ikr}$. [5]

4. a) Starting with the formula derived in class $Y_{\ell\ell}(\theta,\phi) = \frac{(-1)^\ell}{2^\ell \ell!}\left[\frac{2\ell+1}{4\pi}(2\ell!)\right]^{1/2} e^{i\ell\phi}(\sin\theta)^\ell$, obtain the spherical harmonies $Y_{2m}(\theta,\phi), -2 \le m \le 2$. [5]

   b) Show explicitly that the Runge-Lenz vector $\hat{\vec{R}} = \frac{1}{2m}(\hat{\vec{p}}\times\hat{\vec{L}} - \hat{\vec{L}}\times\hat{\vec{p}}) - \frac{e^2}{|\hat{\vec{r}}|}\hat{\vec{r}}$ has components $\hat{R}_\alpha$, that commutes with the Hamiltonian. [5]

5. A hydrogen atom is, at time t = 0, in a state given by the wave function [$\psi_{n\ell m} \rightarrow$ Hydrogen wave fet]

   $$\psi(\vec{r},0) = \frac{1}{\sqrt{10}}\left(2\psi_{100} + \psi_{210} + \sqrt{2}\psi_{211} + \sqrt{3}\psi_{21,-1}\right)$$

   a) Find the expectation value of the Hamiltonian operator in this state. [2]

   b) If the state of a system $\psi(\vec{r},0) = \sum_m C_m \psi_m(\vec{r},0)$, what is the probability of finding the system in the nth state? Hence find the probability of finding the above atom in the state ψ_{211}. [4]

   c) How does $\psi(\vec{r},t)$ change as a function of time? [4]

Unit – II

(*Answer **any two** questions*) [2×10]

6. What is spin-orbit coupling in atomic spectra? How are the states of many electron atom defined? What do you mean by multiplicity of state? State Hund's rule and use it to arrange the energy levels of the first excited state of carbon atom. [2+2+1+1+4]

7. a) Starting from the coupling of two angular momenta, derive an expression for the Lande g-factor and hence the magnetic moment of a monovalent atom. [5+3]

   b) The series limit for the Balmer series of hydrogen atom is 364·0 nm, Determine the wavelength of the first member of Lymen series. [2]

8. a) Derive an expression for the rotational energy eigen states of diatomic molecule. Explain clearly the assumptions behind the calculation. State the selection rules for rotational transitions. Hence show the first four rotational lines in absorption spectra. [3+1+2]

   b) What is the average period of rotation of HCl molecule in J = 1 state? The inter-nuclear distance of HCl is 0·1274 nm and mass of proton = $1{\cdot}66 \times 10^{-27}$Kg. [4]

9. a) State the characteristics of Raman Spectra. What are the advantages of Raman Spectra over infrared spectra? Explain the formation of Raman lines using energy level diagram. Why ant-Strokes lines are less intense than Stokes lines? Explain. [2+1+2+2]

   b) Determine the intensity ratio of the anti-stokes and stokes Raman Lines for a molecular quanta with wave number 50 cm^{-1}. observed at temperature 27°C.

   [Given : h $= 6{\cdot}67 \times 10^{-27}$ args sec, k = $1{\cdot}36 \times 10^{-16}$ erg/k] [3]

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